Eur. Phys. J. B **40**, 295–299 (2004)
DOI: 10.1140/epjb/e2004-00264-0

THE EUROPEAN PHYSICAL JOURNAL B

On a dynamical-like replica-symmetry-breaking scheme for the spin glass

C. De Dominicis[1] and E. Brézin[2,a,b]

[1] Service de Physique Théorique, CE Saclay, 91191 Gif-sur-Yvette, France
[2] Laboratoire de Physique Théorique, École Normale Supérieure, 24 rue Lhomond 75231, Paris Cedex 05, France

Received 19 March 2004 / Received in final form 5 July 2004
Published online 31 August 2004 –

Abstract. Considering the unphysical result obtained in the calculation of the free-energy cost for twisting the boundary conditions in a spin glass, we trace it to the negative multiplicities associated with the Parisi replica-symmetry breaking (RSB). We point out that a distinct RSB, that keeps positive multiplicities, was proposed long ago, in the spirit of an ultra-long time dynamical approach due to Sompolinsky. For an homogeneous bulk system, both RSB schemes are known to yield identical free energies and observables. However, using the dynamical RSB, we have recalculated the twist free energy at the mean-field level. The free-energy cost of this twist is, as expected, positive in that scheme, as it should.

PACS. 64.60.-i General studies of phase transitions

1 Introduction

Lately a rather strange result was uncovered. As is well-known for an $O(N)$ ferromagnet, the breaking of the continuous symmetry and the associated Goldstone modes, tax the forcing of a twist in the orientation of the magnetization (between the $z = 0$ and $z = L$ boundaries) by an amount of free energy proportional to L^{d-2} [1]. Consequently the lower critical dimensions above which symmetry breaking occurs at a non-zero temperature is exactly given by $d_c = 2$. For a spin glass, the broken continuous symmetry group is the reparametrization (or gauge) group. The analog of a rotation twist, is now a (small) gauge twist between the $z = 0$ and $z = L$ boundaries. In contradistinction with the $O(N)$ case, through a long and arduous calculation making use of Parisi's replica symmetry breaking (RSB) [2] on Parisi's truncated Hamiltonian, we obtained a twist free energy cost [3] proportional to $-L^{d-2+\eta}$ (i.e. with a negative coefficient) [4]; the exponent η is the usual order parameter anomalous dimension, computed there to one loop. The implication for the lower critical dimension, namely $d_c = 2 - \eta \simeq 2.5$, was indeed in agreement with previous estimates [5,6]. However the negative sign of the coefficient, i.e. a gain in free energy under twist, was very puzzling. Taking this at face value could point to the instability of a solution with a space homogeneous order parameter. If one considers for instance an Ising antiferromagnet, one may obtain a lower free energy with twisted, i.e. antiparallel boundary conditions. But, given that Parisis's space-homogeneous solution is now proven to give the exact free energy [7,8], this seems unlikely.

A way out of this puzzle seems to be the following. Parisi's solution is (semi) stable, all eigenvalues of the associated Hessian being non-negative when the number of steps R of RSB goes to infinity [9]. However the multiplicities of those eigenvalues are all *negative*. This means that the saddle-point where the free-energy is calculated has the characteristics of a maximum ; hence small excursions away from it will yield an unphysical negative free-energy cost. This situation, in turn, is due to the fact that, in Parisi's RSB, the natural ordering of the box sizes p_u (where u is the discrete overlap) is *reversed*, i.e. one works with

$$p_{R+1} \equiv 1 \geq p_R \geq \cdots \geq p_u \geq \cdots \geq p_1 \geq p_0 \equiv n.$$

In this article we would like to reconsider this question of the free-energy cost under a twist of boundary conditions, in the light of a distinct RSB scheme proposed long ago [10]. This alternative scheme has the following characteristics:

- The box order is *not* reversed, hence one has multiplicities remaining positive:

$$p_0 \gg p_1 \gg \cdots \gg p_u \gg \cdots p_R \gg p_{R+1} \equiv 1.$$

- Whereas p_u in Parisi's approach is a parameter without any physical conjugate field, it is replaced here

[a] e-mail: brezin@corto.lpt.ens.fr
[b] Unité Mixte de Recherche 8549 du Centre National de la Recherche Scientifique et de l'École Normale Supérieure.

by a susceptibility derivative $\dot{\Delta}_u$, a physical quantity associated with a dynamical-like approach to the spin-glass problem (whose history is sketched below). In fact it is a better candidate for an order parameter, since it vanishes above the Almeida-Touless [11] line and is non-zero below.
- At the saddle-point it gives exactly the same value as Parisi's for all observables, including for the overlap probability distribution. It yields also the same eigenvalues for the Hessian at the saddle point [12], but with different individual multiplicities.
- The cost of a twist becomes now *positive*, at least to lowest order (for the "kinetic" part of the free energy, i.e. the part which depends of the spatial variation of the order parameter, since the potential energy is invariant under the twist). The one-loop contribution will undoubtedly take some time to sort out.

To replace in context the choice made here, a short historical reminder seems appropriate. It all started with Sommers [13] proposing a new solution with non-negative entropy to the Sherrington-Kirkpatrick [14] model, a solution possessing a linear response anomaly. Enlarging on the RSB proposal of Blandin et al. [15], soon after, De Dominicis and Garel [16], Bray and Moore [17] linked the non-negativity of the entropy to the infinite limit of what has been called above the box size p_0, a limit that restituted [17] the Sommers solution. Substituting opaque replicas by dynamics [18] led to some physical insight and Sompolinsky [19] proposed to describe spin glasses in terms of the spin correlation function $q = \langle\sigma\sigma\rangle$ and a spin susceptibility, or linear response $\chi = \langle\hat{\sigma}\sigma\rangle$ ($\hat{\sigma}$ being coupled to a magnetic field). The linear response anomaly Δ is closely related to χ. In the long time limit (when the initial time in Langevin's equations is sent to minus infinity) he described the system through an infinite set of relaxation times τ_u, with $u = 1, 2, \cdots R$ and $R \to \infty$. When u remains discrete, his $q(\tau_u)$ is what we call q_u and his $\Delta(\tau_u^{-1})$, our Δ_u (with $-\dot{\Delta}_u = \Delta_u - \Delta_{u+1}$). The approach used in the present article is thus a replica reformulation of Sompolinsky's dynamics as in [10]. At this point one may ask why not retain dynamics and drop replicas altogether. The answer is that

- although some results have been obtained beyond mean-field through dynamics by Sompolinsky and Zippelius [20,21], it is more difficult to work with four time variables than with four replicas. For instance, the spectrum of masses (the eigenvalues of the Hessian) are easily obtained with replicas, but not yet fully sorted out within the dynamics.
- Besides, one is interested in understanding how to obtain a physical answer for the twist free-energy.

In the following we first compute in some details the contribution to the free energy cost of a twist in the parametrization, using Parisi's RSB, as already sketched before [3]. Then we perform the same calculation for the RSB introduced by De Dominicis, Garel and Orland [10] (DGO), that we might call also "dynamical-like" for lack of a better name, getting then the opposite sign. Finally we show that this result may also be understood from the fact that there is a plateau contribution in Parisis's solution, where the overlap fuction remains constant and equal to its Edwards-Anderson value, and does not fluctuate. In the dynamical-like approach there is no such plateau. Hence when considering spatial variations the two approaches give distinct results, whereas they lead to identical results for the bulk mean-field problem.

2 Kinetic free energy à la Parisi

In terms of $n \times n$ matrices q_{ab} ($q_{aa} = 0$) the free energy, in Parisi's truncated model, reads for a spatially homogeneous order parameter

$$nF^{(P)}/L^d = -\sum_{ab}\left(\frac{\tau}{2}q_{ab}^2 + \frac{g}{12}q_{ab}^4\right) + \frac{w}{6}\sum_{abc} q_{ab}q_{bc}q_{ca}. \tag{1}$$

The contribution of the kinetic part of the free energy F_K (i.e. the part which comes form a non-spatially homogeneous order parameter) is given by

$$F_K^{(P)} = \frac{L^{d-1}}{4n}\sum_{i=0}^{L-1}\sum_{a,b}(q_{ab}(i) - q_{ab}(i+1))^2 \tag{2}$$

in which we assume twisted boundary conditions in the z-direction (and periodic boundary conditions in the remaining (d-1) dimensions); space in the z-direction is kept here discrete with $0 \le i \le L$. Using Parisi's overlap function q_u, $u = 0, 1, \cdots, R$ (with $q_{aa} \equiv q_{R+1} = 0$), and the associated box size p_u (with $p_0 \equiv n, p_{R+1} \equiv 1$), one obtains

$$\frac{1}{n}\sum_{ab} q_{ab}^2(i) = \sum_{u=0}^{R+1}(p_u(i) - p_{u+1}(i))q_u^2(i). \tag{3}$$

One needs also

$$\frac{1}{n}\sum_{ab} q_{ab}(i)q_{ab}(i+1) = \sum_{u=0}^{R+1}[\frac{1}{2}(p_u(i) + p_u(i+1)) - \frac{1}{2}(p_{u+1}(i) + p_{u+1}(i+1))]q_u(i)q_u(i+1). \tag{4}$$

This gives then

$$F_K^{(P)} = \frac{L^{d-1}}{4}\sum_{i=0}^{L-1}\sum_{u=0}^{R+1}\big[\big(p_u(i) - p_{u+1}(i)\big)q_u(i) - \big(p_u(i+1) - p_{u+1}(i+1)\big)q_u(i+1)\big][q_u(i) - q_u(i+1)\big]. \tag{5}$$

In the $R \to \infty$ continuum limit, we have

$$p_u(i) - p_{u+1}(i) \to -\dot{p}(u;i)du \tag{6}$$

and finally

$$F_K^{(P)} = -\frac{L^{d-1}}{4}\int_0^L dz \int_0^{1-\epsilon} du \frac{\partial}{\partial z}(\dot{p}(u;z)q(u;z))\frac{\partial}{\partial z}q(u;z). \tag{7}$$

At the bulk saddle-point one has

$$q(u) = \frac{w}{2g}p(u). \tag{8}$$

For twisted boundary conditions, to lowest order in the twist $h(u)$ ($h \ll 1$ and limited to a small support $0 < u < \tilde{x}$),

$$q(u;z) = \frac{w}{2g}\left(u + \frac{z}{L}h(u)\right) + O(h^2) = \frac{w}{2g}p(u;z) + O(h^2). \tag{9}$$

Substituting (9) into (3) we finally obtain

$$F_K^{(P)} = -\frac{1}{8}\left(\frac{w}{2g}\right)^2 L^{d-2}\int_0^{\tilde{x}} du\ h^2(u) \tag{10}$$

that is a *negative* cost in free energy for the introduction of a twist.

3 Kinetic free energy à la DGO

Whereas the zeroth step in Parisi approach is the ($R = 0$) matrix $q_{ab} = q$ for $a \neq b$ (with $q_{aa} = 0$), here the zeroth step is the so-called Sommers starting point (formally identical to the $R = 1$ Parisi's RSB). Namely, one divides the $n \times n$ matrix Q_{ab} into n/p_0 blocks $q_{\alpha,\beta}$ (of size $p_0 \times p_0$) along the diagonal, and $\frac{n}{p_0}(\frac{n}{p_0}-1)$ off-diagonal blocks $r_{\alpha,\beta}$ (of the same size $p_0 \times p_0$). Then one does R steps of RSB on *both matrices* $q_{\alpha,\beta}$ and $r_{\alpha,\beta}$ (whereas in Parisi's approach those steps would only involve the diagonal blocks $q_{\alpha,\beta}$). One thus needs a double labelling for each element of the initial matrix Q_{ab} in order to specify for a (and b) the $p_0 \times p_0$ block matrix under consideration, and the element in this block matrix. One thus writes $a = (\alpha, x)$ with $x = 1, 2, \cdots, n/p_0$ and $\alpha = 1, 2, \cdots, p_0$

$$Q_{ab} \equiv Q_{\alpha,x;\beta,y} = q_{\alpha,\beta}\delta_{x,y} + r_{\alpha,\beta}(1-\delta_{x,y}). \tag{11}$$

The RSB steps apply now to the matrices $q_{\alpha,\beta}$ (with $q_{\alpha,\alpha} = 0$) generating $q_0, q_1, \cdots, q_R, q_{R+1} \equiv 0$, and to the matrices $r_{\alpha,\beta}$ giving rise to $r_0, r_1, \cdots, r_{R+1}$. The successive steps of RSB involve the box sizes $p_0, p_1, \cdots, p_R$ ($p_{R+1} \equiv 1$). In the DGO scheme the box sizes are made to go to infinity in the prescribed natural order

$$p_0 \gg p_1 \gg \cdots \gg p_R \gg p_{R+1} \equiv 1 \tag{12}$$

with at the same time letting the matrix elements of $q - r$ go to zero in such a way that

$$-\dot{\Delta}_0 = p_0(q_0 - r_0) \tag{13}$$

remains finite and in general

$$-\dot{\Delta}_u = p_u[(q_u - r_u) - (p_{u-1} - r_{u-1})] \simeq p_u(q_u - r_u). \tag{14}$$

Here $\dot{\Delta}_u$ can be shown to be the (discrete) susceptibility derivative

$$-\dot{\Delta}_u = \Delta_u - \Delta_{u+1} \tag{15}$$

($-\dot{\Delta}_u$ is a positive quantity).

As a result the Parisi free energy functional $F^{(P)}\{p_u; q_u\}$ is replaced by a dynamical-like free energy $F^{(D)}\{\dot{\Delta}_u; q_u\}$ with the associated stationarity conditions determining $q_u, \dot{\Delta}_u$ and their relationship.

This alternative formulation gives *in the $R \to \infty$ continuum and at the saddle-point* results that are identical to Parisi's. Besides it also leads to the same eigenvalues of the Hessian at the saddle-point [12].

After this brief reminder we now proceed to derive the kinetic contribution:

$$F_K^{(D)} = \frac{L^{d-1}}{4n}\sum_i \sum_{a,b}[Q_{ab}(i) - Q_{ab}(i+1)]^2. \tag{16}$$

We have:

$$\frac{1}{n}\sum_{ab} Q_{ab}^2 = \frac{1}{n}\sum_{\alpha,\beta}\sum_{x,y}[(q_{\alpha,\beta} - r_{\alpha,\beta})\delta_{x,y} + r_{\alpha,\beta}]^2. \tag{17}$$

Summing upon x, y one obtains

$$\frac{1}{n}\sum_{ab} Q_{ab}^2 = \frac{1}{n}\sum_{\alpha,\beta}\left\{\frac{n}{p_0}[(q_{\alpha,\beta} - r_{\alpha,\beta})^2 + 2r_{\alpha,\beta}(q_{\alpha,\beta} - r_{\alpha,\beta})] + \left(\frac{n}{p_0}\right)^2 r_{\alpha,\beta}^2\right\}. \tag{18}$$

After R steps of RSB one obtains

$$\begin{aligned}\frac{1}{n}\sum_{ab} Q_{ab}^2 &= \frac{1}{n}\frac{n}{p_0}p_0\sum_{u=0}^{R}(p_u - p_{u+1})\left[\left(q_u^2 - r_u^2\right) + \frac{n}{p_0}r_u\right]\\ &= \sum_{u=0}^{R} p_u\left[\left(q_u^2 - r_u^2\right) - \left(q_{u-1}^2 - r_{u-1}^2\right)\right] - r_{R+1}^2.\end{aligned} \tag{19}$$

Since $p_u \to \infty$, $\frac{p_u}{p_{u-1}} \to 0$, $q_u - r_u \sim \frac{1}{p_u}$ for $u = 1, \cdots, R$ in the limit (12) the equation (19) reduces to

$$\frac{1}{n}\sum_{ab} Q_{ab}^2 = -2\sum_{u=0}^{R} q_u\dot{\Delta}_u - r_{R+1}^2. \tag{20}$$

The analog of (3) is now

$$\frac{1}{n}\sum_{ab} Q_{ab}^2(i) = -2\sum_{u=0}^{R} q_u(i)\dot{\Delta}_u(i) - r_{R+1}^2. \tag{21}$$

Likewise (4) becomes

$$\frac{1}{n}\sum_{ab} Q_{ab}(i)Q_{ab}(i+1) = \sum_{u=0}^{R} p_u[q_u(i)q_u(i+1) - r_u(i)r_u(i+1)] - r_{R+1}^2. \tag{22}$$

The box sizes going to infinity do not carry a space index any more and we end up with

$$\frac{1}{n}\sum_{ab} Q_{ab}(i)Q_{ab}(i+1) = \sum_{u=0}^{R}\Big[\dot{\Delta}_u(i)q_u(i+1) + \dot{\Delta}_u(i+1)q_u(i)\Big] - r_{R+1}^2. \quad (23)$$

The result for the kinetic part of the mean field free energy functional is thus

$$\begin{aligned} F_K^{(D)} &= \frac{L^{d-1}}{4n}\sum_{i=0}^{L-1}\sum_{ab}\Big(Q_{ab}(i)-Q_{ab}(i+1)\Big)^2 \\ &= -\frac{L^{d-1}}{2}\sum_{i=0}^{L}\sum_{u=0}^{R}\Big[q_u(i)\dot{\Delta}_u(i)+q_u(i+1)\dot{\Delta}_u(i+1) \\ &\quad - q_u(i)\dot{\Delta}_u(i+1) - q_u(i+1)\dot{\Delta}_u(i)\Big] \\ &= -\frac{L^{d-1}}{2}\sum_{i=0}^{L}\sum_{u=0}^{R}\Big[\Big(q_u(i)-q_u(i+1)\Big) \\ &\quad \times\Big(\dot{\Delta}_u(i)-\dot{\Delta}_u(i+1)\Big)\Big] \end{aligned} \quad (24)$$

and in the double continuum limit, in which both space is continuous and R, the number of steps of RSB, goes to infinity

$$F_K^{(D)} = -\frac{L^{d-1}}{2}\int_0^L dz\int_0^{x_1} du\frac{\partial q(u;z)}{\partial z}\frac{\partial\dot{\Delta}(u;z)}{\partial z}. \quad (25)$$

At the saddle-point one has [10,19]

$$-\dot{\Delta}(u) = \frac{2g}{w}q(u)\dot{q}(u) \quad (26)$$

which is the "anomalous fluctuation-dissipation relationship" for the spin-glass. Again, to lowest order in the twist h(t), one can just keep, as in (9)

$$q(u;z) = \frac{w}{2g}[u+\frac{z}{L}h(u)] + O(h^2) \quad (27)$$

from which one obtains, to lowest order, the free energy for that twist

$$F_K^{(D)} = \frac{1}{4}(\frac{w}{2g})^2L^{d-2}\int_0^{\tilde{x}} du\; h^2(u) \quad (28)$$

so that

$$F_K^{(D)} = -2F_K^{(P)}. \quad (29)$$

The signs are opposite: the free energy cost under twist is *positive* for the dynamical-like RSB.

4 A simple calculation at the saddle-point

Given the previous difference between the two RSB schemes when one considers the spatial variation of the order parameter, one may examine in the light of the previous calculation why the two free energies happen to coincide for a spatially uniform order parameter.

For instance let us consider the bulk contribution to the free energy per unit volume which is quadratic in the order parameter:

$$f_\tau = \frac{\tau}{4}\sum_{ab}q_{ab}^2. \quad (30)$$

In Parisi's scheme one finds

$$\begin{aligned} f_\tau^{(P)} &= \frac{\tau}{4}\sum_{u=0}^{R-1}(p_u-p_{u-1})q_u^2 + (p_R-1)q_R^2 \\ &= \frac{\tau}{4}\Big[-\int_0^{x_1} duq^2(u) + (x_1-1)q^2(x_1)\Big] \\ &= \frac{\tau}{4}\Big(\frac{w}{2g}\Big)^2\Big[-\frac{x_1^3}{3}+(x_1-1)x_1^2\Big]. \end{aligned} \quad (31)$$

Here the second term in the bracket comes from the "plateau" sector of $q(u)$ ($x_1 \le u < 1$), at which it remains fixed to its Edwards-Anderson value

$$q(x_1) = \frac{w}{2g}x_1. \quad (32)$$

The mean field stationarity conditions determine x_1 in terms of the external parameters (temperature and coupling constants).

For the dynamical-like approach one finds

$$\begin{aligned} f_\tau^{(D)} &= \frac{\tau}{4}[-\sum_{u=0}^{R}2q(u)\dot{\Delta}(u) - r_{R+1}^2 \\ &= \frac{\tau}{4}\Big[2\int_{u=0}^{x_1} duq^2(u)\frac{2g}{w}\dot{q}(u) - q^2(x_1)\Big] \\ &= \frac{\tau}{4}\Big(\frac{w}{2g}\Big)^2\Big[\frac{2x_1^3}{3}-x_1^2\Big]. \end{aligned} \quad (33)$$

The two results (31) and (33) at the end coincide as announced, however the origin of the various terms are different. Indeed in (31) part of the answer comes from the plateau where fluctuations are not allowed under twist. Thus when considering fluctuations the plateau part in (31) will not contribute and one will obtain

$$\Big(f_\tau^{(D)}\Big)_{fluct.} = -2\Big(f_\tau^{(P)}\Big)_{fluct.} \quad (34)$$

as we found in (29).

Similar calculations for the parts of the free energy which are cubic and quartic in the order parameter would show a similar conspiracy to make the bulk contributions identical, but yield different results when the order parameter varies in space.

5 Conclusion

We have thus demonstrated that two approaches, using different RSB schemes, that provide identical results for the bulk mean-field energy, lead to distinct (and opposite) answers when one enforces spatial variations of the order parameter. The negative multiplicities occuring in Parisi's scheme were responsible for a decrease of the free energy under twist, whereas the dynamical-like DGO scheme, that enjoys positive multiplicities, does lead to an increase under twist.

At this point many questions remain opened. First, and even without introducing constraints in the boundary conditions, one would like to compare the fluctuations for the bulk. As we know such loop corrections need be included to describe physics below six dimensions and one would like to see whether such fluctuations are identical to, or differ from, fluctuations calculated within Parisi's approach in finite dimensions.

Next one would like to compute responses associated with inhomogeneous boundary conditions (to one-loop) as was done within Parisi's approach in [3]. Finally it would be very gratifying to construct a direct connection between the dynamical field theoretic approach of Sompolinsky and Zippelius [20,21] and the dynamical-like replica approach considered here.

We would like to thank G. Parisi and S. Franz for useful discussions. One of us (CDD) would like to thank also G. Biroli and A. Crisanti.

References

1. E. Brézin, C. De Dominicis, Eur. Phys. J. B **24**, 353 (2001)
2. G. Parisi, Phys. Rev. Lett. **43**, 1754 (1979); G. Parisi, J. Phys. A **13**, L115 (1980); G. Parisi, J. Phys. A **13**, 1101 (1980); G. Parisi, J. Phys. A **13**, 1887 (1980)
3. E. Brézin, C. De Dominicis, Eur. Phys. J. B **30**, 71 (2002)
4. T. Aspelmeier, M.A. Moore, A.P. Young, Phys. Rev. Lett. **90**, 127202 (2003). In this article the authors have computed the free-energy cost of a change from periodic to antiperiodic boundary conditions. This cost has no simple connection with the gauge twist cost of [3]
5. N. Kawashima, A.P. Young, Phys. Rev. B **53**, R484 (1996)
6. S. Franz, G. Parisi, M. Virasoro, J. Phys. I France **4**, 657 (1994)
7. F. Guerra, Comm. Math. Phys. **233**, 1 (2003)
8. M. Talagrand, C. R. Acad. Sc. Paris **337**, 111 (2003)
9. C. De Dominicis, I. Kondor, T. Temesvari, in *Spin glasses and random fields*, edited by A.P. Young (World Scientific, Singapore, 1998)
10. C. De Dominicis, M. Gabay, H. Orland, J. Phys. Lett. **42**, L523 (1981)
11. J.R. de Almeida, D.J. Thouless, J. Phys. A **11**, 983 (1978)
12. I. Kondor, C. De Dominicis, J. Phys. A **16**, L73 (1983)
13. H.J. Sommers, Z. Phys. B **31**, 301 (1978); H.J. Sommers, Z. Phys. B **33**, 173 (1979)
14. D. Sherrington, S. Kirkpatrick, Phys. Rev. Lett. **35**, 1792 (1975)
15. A. Blandin, J. Phys. Colloq. France **39**, C6-1499 (1978); A. Blandin, M. Gabay, T. Garel, J. Phys. C **13**, 403 (1980)
16. C. De Dominicis, T. Garel, J. Phys. Lett. France **40**, 575 (1979)
17. A. Bray, M. Moore, J. Phys. C **13**, 419 (1980)
18. C. De Dominicis, Phys. Rev. B **18**, 4913 4913
19. H. Sompolinsky, Phys. Rev. Lett. **47**, 935 (1981)
20. H. Sompolinsky, A. Zippelius, Phys. Rev. Lett. **50**, 1297 (1983)
21. A. Zippelius, Phys. Rev. B **29**, 2717 (1984)